Investigation of Longitudinal Control System for a Small Hydrofoil Boat

William H. Phillips* and John D. Shaughnessy†
NASA Langley Research Center, Hampton, Va.

An analysis of a hydromechanical system for longitudinal control of a small hydrofoil boat is presented. The system incorporates height and acceleration sensors operating flaps on the foils through a mechanical linkage. Effects of some of the system parameters on the stability and response to waves are shown. The results indicate that the system is capable of providing adequate stability, but the response to stern waves at low frequencies is larger than desired.

Nomenclature

A = gearing ratio between elevator angle and flap angle, δ_e/δ_f
a = wave amplitude
b_f = span of front foil
c_f = chord of front foil
C_L = lift coefficient, $L/\frac{1}{2}\rho V^2 S_f$
C_m = pitching-moment coefficient, $M/\frac{1}{2}\rho V^2 S_f c_f$
g = acceleration of gravity
H = hinge moment
H_z = hinge moment due to vertical displacement
$H_{\ddot{z}_m}$ = hinge moment due to bobweight vertical acceleration
H_{δ_f} = flap hinge moment
I_f = moment of inertia of flap system
K = gearing ratio between bobweight arm angle and flap angle, δ_m/δ_f
K_f = radius of gyration of flap
K_y = radius of gyration of boat
L = lift
ℓ = distance from bobweight center of mass to its pivot point
M = pitching moment
m = mass of boat
m_f = mass of flap
m' = mass of bobweight
P = period of oscillation
q = dynamic pressure, $\frac{1}{2}\rho V^2$
R = ratio of wave velocity to boat velocity (positive for stern waves and negative for head waves)
S_f = area of front foil
$T_{1/2}$ = time to damp to one-half amplitude
V = boat velocity
V_w = wave velocity (positive for stern waves and negative for head waves)
x = horizontal displacement, positive forward
x_f = horizontal distance from front foil hydrodynamic center to boat center of gravity
x_p = horizontal distance from bobweight pivot point to boat center of gravity
x_r = horizontal distance from hydrodynamic center of rear foil to boat center of gravity
x_w, z_w = coordinates of particle of water near surface in sinusoidal wave of small amplitude
z_m = vertical displacement of bobweight
z_0 = vertical displacement of boat center of gravity, positive downward
z_{w0} = wave height amplitude
z_w = height of water above mean water surface at center of gravity of boat, positive upward.
α = total angle of attack at front foil
α_{w0} = amplitude of angle of attack due to waves
α_0 = angle of attack of boat due to vertical velocity, $\dot{z}_0/V$
α_w = angle of attack due to vertical velocity of water, $\dot{z}_w/V$
δ_e = elevator angle
δ_f = flap angle
δ_m = bobweight arm angle
θ = pitch angle of boat
ω_A = actual frequency felt by boat in encountering waves
ω_B = fictitious frequency which would be felt by boat in riding fixed wavy surface with same wavelength as waves on water.
λ = wavelength of waves
ϕ_f = phase angle that wave motion affecting front foil leads that at center of gravity
ϕ_r = phase angle that wave motion affecting rear foil lags that at center of gravity
σ = wave frequency measured at fixed point
ρ = water density

Received December 20, 1974; revision received May 12, 1975.
Index categories: Marine Vessel Trajectories, Stability, and Control; Marine Hydrodynamics, Vessel and Control Surface.
*Chief, Flight Dynamics & Control Division. Associate Fellow AIAA.
†Aero-Space Technologist, Stability & Control Branch, Flight Dynamics & Control Division.

Introduction

To achieve inherent stability, most hydrofoil boats, particularly small ones, utilize surface-piercing foils. Improved efficiency can be obtained with submerged foils, but in this case height control must be provided by means of an automatic control system.

Previous analyses of the longitudinal stability of hydrofoil boats have included studies of stability and response to waves of an uncontrolled boat,[1-3] and studies of boats with rather complex automatic control systems, using electronic sensors and electrohydraulic servomechanisms to operate controls on the foils.[4-6] The object of the present study is to analyze a relatively simple longitudinal control system applicable to a boat with submerged foils. The control system utilizes sensors which required no power source and which have sufficient force output to operate control surfaces on the foils directly through a mechanical linkage, thereby avoiding the need for servomechanisms. The stability and response characteristics of the system are analyzed by means of root locus studies, studies of transient response to control input, and frequency response studies. The results are limited to analysis of a linearized system and are, therefore, applicable to small disturbances.

Description of Longitudinal Control System

A sketch of the longitudinal control system under consideration is shown in Fig. 1. The height sensor is shown conceptually as a pivoted strut extending into the water. The drag force of the strut is balanced against the hydrodynamic hinge moment of a flap on the main hydrofoil, so that the flap deflects down with increasing submergence of the strut. (A more practical arrangement might be a planing surface mounted at the end of a pivoted strut to follow the water surface. The strut would then drive the flap linkage through a spring). A trailing-edge tab on the flap is used to trim the strut moment at the desired depth. A bobweight is also linked to the flap in such a way that upward acceleration of the boat causes the flap to move up. The static moment of the bobweight is balanced by an approximately constant-tension spring, and its motion is resisted by the damping and restoring moments on the flap, and by a dashpot if additional damping is required.‡

The reasoning behind the design of this control system is as follows: A hydrofoil boat with submerged foils has little or no restoring force tending to keep it at desired height above the water. A control system to sense the height and operate a control surface to restore the boat to its equilibrium height is, therefore, essential. If the controls were operated only as a function of height, however, the resulting restoring force as a function of height might tend to produce an undamped oscillation about the trim position and any lag in the control motion due to inertia or damping would produce a force in phase with the vertical velocity, possibly resulting in an unstable oscillation.

The bobweight is intended to restore the damping of the motion of the boat by supplying some phase lead in the flap motion. The bobweight provides a force opposing the vertical acceleration of the boat, but any lag in the flap motion caused by inertia or damping in the system provides a component of flap motion opposing the vertical velocity. Also, by locating the bobweight ahead of the center of gravity, some control motion to oppose pitching velocity may be obtained. The effects of the bobweight might also be expected to reduce the vertical response of the boat to waves. In some of the subsequent analytical studies, the effect of linking an elevator on the rear foil to move in phase with the flap on the main foil is also considered.

Analysis

Equations of motions of boat

The equations of motion are derived using the same notation as is conventionally used in airplane stability theory. The notation of the present report is based on that of Ref. 7. The derivation of these equations, as applied to hydrofoil boats, and evaluation of the stability derivatives, is given in Ref. 2. Some of the notation used is given in Fig. 1. Because the main object of this study is to determine the effect of the control system, some simplifications are made in the basic equations of the boat. The flap on the front foil is assumed to produce vertical force and no pitching moment; the flap (or elevator) on the rear foil is assumed to produce pitching moment and no lift. The effect of downwash from the front foil on the rear foil, which would, in the linearized stability theory, introduce forces and moments proportional to rate of change of angle of attack, has been neglected. The equations of motion of the boat based on these assumptions may be writtern as follows:

$$mV(\dot{\alpha}_0-\dot{\theta})/\rho S_f-\alpha_0 C_{L_\alpha}-(\dot{\theta}c_f/2V)C_{L_q} = \alpha_{w_f}C_{L_{\alpha_f}}+\alpha_{w_r}C_{L_{\alpha_r}}+\delta_f C_{L_{\delta_f}} \quad (1)$$

$$mK_y^2\ddot{\theta}/\rho S_f c_f-\alpha_0 C_{m_\alpha}-(\dot{\theta}c_f/2V)C_{m_q} = \alpha_{w_f}C_{m_{\alpha_f}}+\alpha_{w_r}C_{m_{\alpha_r}}+\delta_e C_{m_{\delta_e}} \quad (2)$$

‡Several hydrofoil boats incorporating a system similar to that studied have been built by J.K. Roper of Hancock, N.H. Information on experience with these boats is not available to the authors.

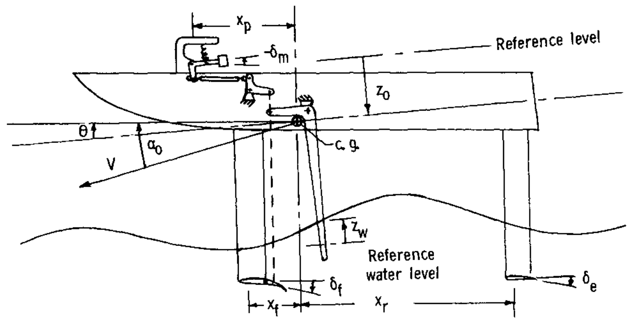

Fig. 1 Control system and definition of symbols used in analysis.

Equation of control system

The equation of the control system is derived under the assumption that all moments of inertia and hinge moments are referred to motion of the flap about its hinge line. The flap equation is

$$I_f\ddot{\delta}_f-\delta_f H_{\delta_f}-\dot{\delta}_f H_{\dot{\delta}_f}-(z_w+z_0)H_z-\ddot{z}_m H_{\ddot{z}_m}=0$$

but $\ddot{z}_m=\ddot{z}_0-x_p\ddot{\theta}+\ell\ddot{\delta}_m$

let $\delta_m=K\delta_f$

hence, $\ddot{z}_m=\ddot{z}_0-x_p\ddot{\theta}+\ell K\ddot{\delta}_f$

the flap equation therefore becomes

$$\ddot{\delta}_f(I_f-\ell K H_{\ddot{z}_m})-\delta_f H_{\delta_f}-\dot{\delta}_f H_{\dot{\delta}_f} -(z_w+z_0)H_z-(\ddot{z}_0-x_p\ddot{\theta})H_{\ddot{z}_m}=0 \quad (3)$$

In this equation, the term I_f, the moment of inertia of the flap system, includes the effect of the flap itself plus all linkages and the bobweight. If the main contributors to this inertia are the flap and the bobweight,

$$I_f=m_f K_f^2+K^2 m'\ell^2$$

Calculation of effect of waves

The response to sinusoidal wave inputs is calculated for two cases, one in which the waves are traveling in the opposite direction from the boat (head waves) and one in which the waves are traveling in the same direction (stern waves). The waves influence the boat through the effects of vertical and horizontal components of velocity at the front and rear foils and through their influence on the height sensor in the control system.

For a boat operating at low values of lift coefficient, the horizontal components of wave velocity have a relatively small effect on the lift of the foils. These components are neglected in the present analysis. The vertical components of velocity are taken into account as an angle of attack disturbance α_w equal to $\dot{z}_w/V$ acting at the front and rear foils.

To determine the expression for the angles of attack at the front and rear foils and the instantaneous value of wave height at the height sensor during passage of the boat over sinusoidal waves, some formulas based on the theory of wave motion are required. The position of a particle of water near the surface in a sinusoidal wave of small amplitude is given by the expressions (Ref. 8)

$$x_w=a\cos[(2\pi/\lambda)x-\sigma t]$$

$$z_w=a\sin[(2\pi/\lambda)x-\sigma t]$$

where $\sigma=2\pi V_w/\lambda$

The vertical velocity of a water particle is, therefore

$$\dot{z}_w = -a\sigma\cos[(2\pi/\lambda)x - \sigma t]$$

let

$$\alpha_w = \dot{z}_w / V$$

then

$$\alpha_w = \alpha_{w_0}\cos[(2\pi/\lambda)x - \sigma t]$$

where

$$\alpha_{w_0} = -a\sigma/V$$

In the present analysis, the wave amplitude is defined by the value of α_{w_0}. For a given value of α_{w_0}, waves with different wavelengths λ have differing heights but the same slope or the same amplitude of vertical velocity of the water particles. The ratio of the amplitude of wave height z_{w_0} to α_{w_0} is

$$z_{w_0}/\alpha_{w_0} = a/a\sigma/V) = V/\sigma$$

Hence, the wave height is related to α_{w_0} by the formula

$$z_{w_0} = (V/\sigma)\alpha_{w_0}$$

The phase angle of the waves is defined with respect to the position of the center of gravity of the boat. If the front foil is ahead of the center of gravity, the wave motion affecting the front foil will lead that at the center of gravity by the phase angle

$$\phi_f = 2\pi x_f/\lambda$$

Likewise, the wave at the rear foil will lag that at the center of gravity by the phase angle

$$\phi_r = -2\pi x_r/\lambda$$

The actual frequency felt by the boat in encountering the waves is

$$\omega_A = \omega_B(1 - V_w/V) = \omega_B(1-R) \quad (4)$$

where $\omega_B = 2\pi V/\lambda$ is a fictitious frequency which would be encountered by the boat in riding a fixed, wavy surface with the same wavelength as the waves. The wave velocity V_w is considered negative for head waves and positive for stern waves.

The time variation of the angle of attack due to the waves at the center of gravity of the boat may, therefore, be written

$$\alpha_w = \alpha_{w_0}\cos\omega_A t$$

The angle of attack at the front foil is then

$$\alpha_{w_f} = \alpha_{w_0}\cos(\omega_A t + 2\pi x_f/\lambda]$$

expanding this expression

$$\alpha_{w_f} = \alpha_{w_0}\left[\cos\frac{2\pi x_f}{\lambda}\cos\omega_A t - \sin\frac{2\pi x_f}{\lambda}\sin\omega_A t\right]$$

a similar expression may be written for the angle of attack at the rear foil. The time variation of the height of the water surface at the center of gravity of the boat is given by the expression

$$z_w = \frac{V}{\sigma}\alpha_{w_0}\sin\omega_A t = \frac{V}{V_w}\frac{\lambda}{2\pi}\sin\omega_A t = \frac{R\lambda}{2\pi}\sin\omega_A t$$

To use these expressions, the value of the wavelength λ and the wave velocity V_w must be related to ω_A. This relation is based on the assumption of waves in deep water, that is, water that is deep compared to the wavelength. The wave velocity based on this assumption is reasonable accurate for water deeper than half the wavelength. The wave velocity is

$$V_w = (g\lambda/2\pi)^{1/2}$$

Substituting this expression in Eq. (4) and solving for ω_B yields the expression

$$\omega_B = \omega_A + g/2V \pm [(\omega_A + g/2V)^2 - \omega_A^2]^{1/2}$$

Utilizing the relations

$$V_w = V(1 - \omega_A/\omega_B)$$

and

$$\lambda = 2\pi V/\omega_B$$

the expressions for z_w, α_{w_f}, and α_{w_r} may be placed entirely in terms of ω_A and the known velocity and dimensions of the boat.

The frequency response of the boat is calculated by applying simultaneously the expressions for α_{w_f}, α_{w_r}, and z_w multiplied by their appropriate coefficients, in Eqs. (1), (2), and (3). The calculation is carried out numerically on a high-speed digital computer, using complex arithmetic, with the cosine and sine components of the forcing terms being taken as the real and imaginary parts of a complex exponential function. The real and imaginary parts (or amplitude and phase angle) of each state variable describing the response of the boat are printed out as functions of the frequency ω_A.

Selection of Parameters for Study

The dimensions, weight, and speed of the boat used in the study are representative of a small runabout. These values are given in Table 1.

Expressions for the stability derivatives in terms of these dimensions are given in detail in Ref. 2. For the present study, only the major terms in these expressions were retained in calculating the derivatives. The values of the derivatives used are given in Table 2.

Inasmuch as the authors have had no previous experience with control systems of the type under consideration, a set of initial parameters of the flap system was selected, based on somewhat arbitrary physical reasoning. This set of parameters is referred to as the "standard case" and the effects of variations of various parameters from this condition are investigated subsequently. The calculations are for a boat speed relative to the water of 7.62 m/sec (25 fps) or a dynamic pressure of 29,000 Pa (606 psf), corresponding to a lift coefficient of 0.3. The parameters for the standard case are selected as follows: The flap restoring moment was assumed to come entirely from the hydrodynamic moments on the flap. Assuming $C_{h_{\delta_f}} = 0.01$ per deg the restoring moment is

$$H_{\delta_f} = C_{h_{\delta_f}} q b_f c_f^2 = 1.45\text{N-m/deg}\,(1.07\text{ ft-lb/deg})$$

For the flap to work on its linear range of operation for the maximum excursions of the boat, it is assumed that 0.61 m (2ft) of vertical displacement of the boat gives full flap deflection of 20°. The value of H_z is, therefore, 47.5 N-m/m (10.7 ft-lb/ft). The bobweight effectiveness was arbitrarily selected so that 1 g of vertical acceleration at the bobweight location would produce the maximum flap deflection of 20°. Assuming a bobweight arm of 0.305 m (1 ft), and a gearing K of -1.0, the mass m' of the bobweight is 9.7 kg (0.663 slug). Assuming a flap mass of 4.536 kg (0.31 slug) and a flap radius of gyration of 0.03 m (0.1 ft), the value of I_f is 0.905 kg-m^2(0.667 slug-ft^2). Hence, almost all the inertia of the flap system comes from the bobweight. If all the mass unbalance in the system comes from the bobweight

$$H_{\ddot{z}_m} = -m'\ell K = 2.95\ \frac{\text{N}-\text{m}}{\text{m/sec}^2}\ \left[0.664\ \frac{\text{ft-lb}}{\text{ft/sec}^2}\right]$$

Table 1 Characteristics of boat used in study

Parameter	Value	
w	5337 N	(1200 lb)
m	544 kg	(37.3 slugs)
ρ	999.8 kg/m^3	(1.94 slugs/ft^3)
S_f	0.613 m^2	(6.60 ft^2)
b_f	1.914 m	(6.28 ft)
c_f	0.320 m	(1.05 ft)
K_y	1.92 m	(6.30 ft)
S_r	0.245 m^2	(2.64 ft^2)
x_f	0.6096 m	(2 ft)
x_r	3.048 m	(10 ft)
z_f	1.828 m	(6 ft)
z_r	1.828 m	(6 ft)
ℓ	0.3048 m	(1 ft)
m'	9.68 kg	(0.663 stugs)
K	−1.0	
m_f	6.52 kg	(0.310 slugs)
K_f	0.0305 m	(0.1 ft)
x_p	1.219 m	(4 ft)

Table 2 Stability derivatives of boat used in study (per rad)

Parameter	Value
C_{L_α}	−5.04
C_{L_q}	−15.24
$C_{L_{\delta_f}}$	−1.00
C_{m_α}	−6.94
C_{m_q}	−179
C_{m_δ}	−3.81
$C_{h_{\delta_f}}$	-0.573

The value of $H_{\dot{\delta}_f}$ was arbitrarily selected as 3.88 N-m/(rad/sec)[2.86 ft-lb/(rad/sec)], which gives about 0.3 times critical damping of the flap system acting as a single-degree-of-freedom system. The hydrodynamic damping of the flap is estimated to be 2.98 N-m/(rad/sec) [2.2 ft-lb/(rad/dec)]. There may, therefore, be no need for the addition of a dashpot to supply the desired damping.

A forward position of the bobweight is considered advantageous. The value of x_p was selected arbitrarily as 1.22 m (4 ft). The values of these initial parameters serve only as a reasonable reference for comparison of the effects of various changes. The effects of varying many of the parameters are presented subsequently.

Results

Results are shown in terms of root locus plots as various control system and boat parameters are varied, transient responses to various disturbances, and frequency response plots for sinusoidal wave inputs.

The roots of the characteristic equation for the standard case, and the corresponding values of period and time to damp to one-half amplitude are:

Root $= -26.3$, $T_{1/2} = 0.0264$ sec

Root $= -9.77$, $T_{1/2} = 0.0709$ sec

Root $= -5.18 \pm 10.6i$, $P = 0.59$ sec, $T_{1/2} = 0.134$ sec

Root $= -0.408 \pm .879i$, $P = 7.15$ sec, $T_{1/2} = 1.7$ sec

The two real roots represent subsidences so rapid as to be of little concern. The two oscillatory modes have widely differing periods but each is well damped.

The static resistance of the boat to changes in height above the water surface is directly related to the parameter H_z. The effect of varying the value of H_z on the roots with other parameters at their standard values is shown in Fig. 2. As H_z is increased from the initial value of 47.5 N-m/m, the longer-period oscillatory mode becomes better damped, eventually turning into a pair of subsidences at H_z of about 156 N-m/m. With further increase in the value of H_z, another oscillatory mode of higher frequency appears. Both this and the original high-frequency mode retain good damping up to a value of H_z of 222 N-m/m (50 ft-lb/ft).

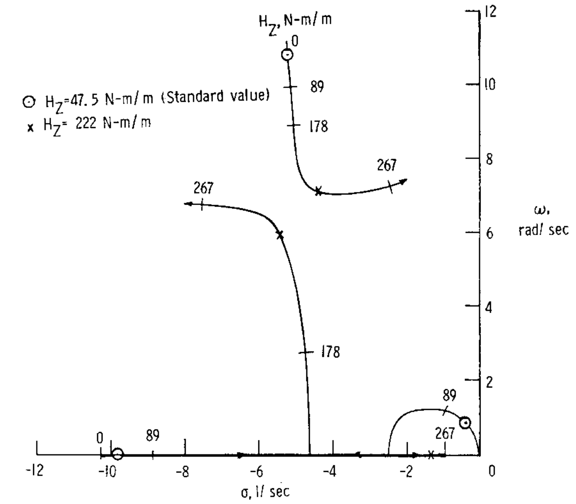

Fig. 2 Effect on roots of varying H_z, variation of flap hinge moment with height (1 lb = 4.448 N).

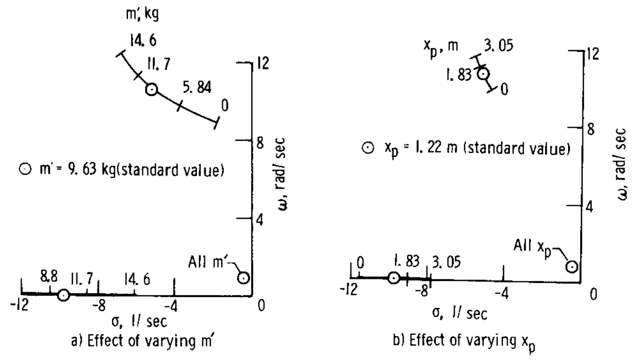

Fig. 3 Effect on roots of varying bobweight mass m' and bobweight pivot location x_p. H_z = 47.5 N-m/m (10.7 ft-lb/ft) (1 kg = 0.0685 slug; 1 m = 3.28 ft).

The effects of varying the bobweight mass m' and the bobweight pivot location x_p are shown in Fig. 3. For these cases, the inertia about the flap hinge was kept constant. Without the bobweight, the high-frequency oscillatory mode would be poorly damped. Increasing the bobweight mass to the standard value effectively improves the damping of this mode. Changing the mass of the bobweight has a negligible effect on the low-frequency oscillatory mode, however. The effect of the bobweight location (Fig. 4b) is practically negligible with the other parameters at their standard values.

Inasmuch as the dynamic stability of the boat as indicated by the roots appears satisfactory for a value of H_z as high as 222 N-m/m, the effects of varying the bobweight mass m' and the pivot location x_p were studied for this high value of H_z. The moment of inertia of the flap system I_f was kept constant as the bobweight mass was varied. This condition is probably unrealistic because most of the inertia of the flap system comes from the bobweight. These results (Fig. 4) show that, as the bobweight mass is reduced, one high-frequency oscillatory mode becomes poorly damped. The standard bobweight is about right to provide good damping of both modes, but a larger bobweight would result in decreased damping of the other high-frequency mode. Likewise, the bobweight location x_p of 1.22 m appears to be about right to distribute

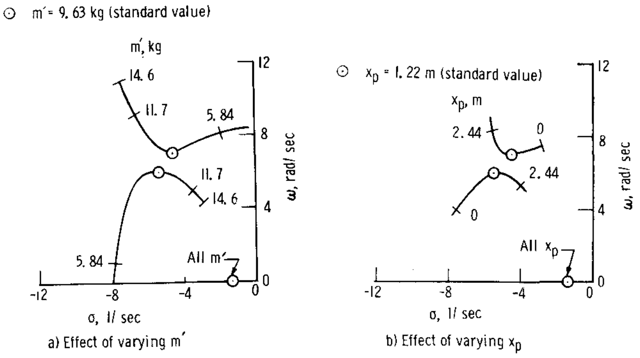

Fig. 4 Effect on roots of varying bobweight mass m' and bobweight pivot location x_p. $H_z = 222$ N-m/m (50 ft-lb/ft), other variables standard (1 kg = 0.0685 slug: 1 m = 3.28 ft).

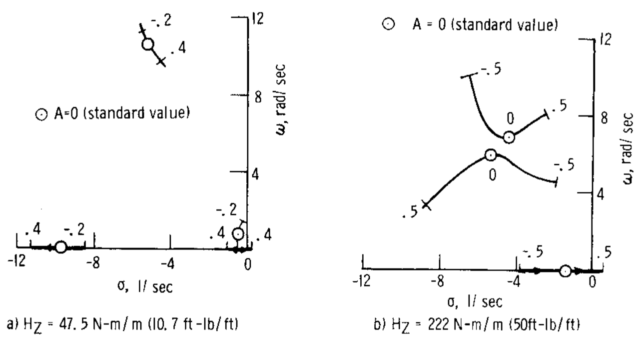

Fig. 5 Effect on roots of elevator gearing with two values of H_z.

the damping equally between the two modes. In all cases, however, a real root corresponding to a subsidence with a time constnt of about 4.7 sec is present which is practically unaffected by the presence of the bobweight.

In all cases presented thus far, the flap has been assumed to produce lift but no pitching moment about the center of gravity of the boat. A pitching moment proportional to flap deflection may be obtained by gearing the elevator to the flap. This gearing ratio A is considered positive when the elevator moves in the same direction as the flap, and negative when it moves in the opposite direction. The effect of this elevator gearing on the roots is shown in Fig. 5 for values of H_z of 47.5 and 222 N-m/m, with all other parameters at their standard values. With H_z of 47.5 N-m/m, increasing A positively causes the long-period oscillatory mode to change to a pair of subsidences, one of which becomes unstable at a value of A greater than 0.5. With H_z of 222 N-m/m, the real root also becomes unstable at about the same value of gearing. The damping of the oscillatory modes in this case is distributed unequally as the value of A is varied from zero. These results indicate that a positive value of the elevator gearing A, which might be thought to reduce pitching response to waves, must be used with care because it slows the transient response of the boat. If the positive value of A is too large, it will produce static instability.

The root loci presented previously provide a convenient survey of the effects of a number of parameters, but transient responses to various disturbances are desirable to examine the response characteristics of the boat in more detail. Transient responses showing the variations of angle of attack, pitching velocity, pitch angle, height, and flap deflection, assuming release from an initial condition of pitch angle, flap deflection, and height, are shown in Figs. 6 and 7 for values of H_z of 47.5 and 222 N-m/m, respectively. These results show that the responses appear desirably smooth and well damped. The long-period mode is excited by the initial condition of pitch angle, whereas the short-period oscillatory mode, which involves mainly an oscillation of the flap system, is excited by

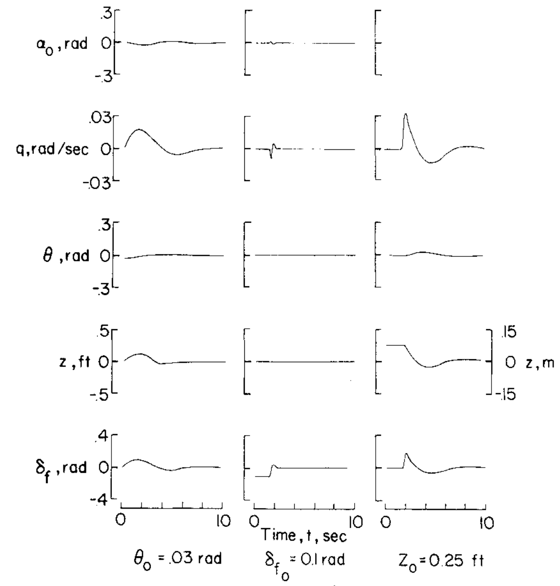

Fig. 6 Transient responses, standard case, $H_z = 47.5$ N-m/m (10.7 ft-lb/ft), other variables standard.

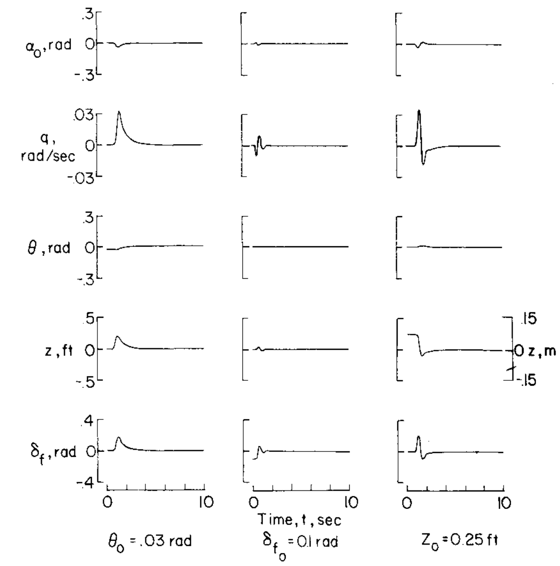

Fig. 7 Transient responses, $H_z = 222$ N-m/m (50 ft-lb/ft), other variables standard.

an initial condition of flap deflection. The initial condition of height excites both modes to some extent.

The frequency response of the boat to head waves and stern waves is shown in Fig. 8 for the case $H_z = 47.5$ N-m/m and in Fig. 9 for the case $H_z = 222$ N-m/m. The wave disturbance was assumed to have a constant value of α_{w_0} of 0.01 rad, or amplitude of the vertical velocity of the water particles of 0.076 m/sec (0.25 fps) as the frequency was varied. The frequency was varied by changing the wavelength while keeping the boat speed constant. As a result of these assumptions, the wave height varies with frequency. The wave height z_w is shown as a function of frequency for comparison with the vertical response of the boat. The values of α_0 and z_0, the angle of attack and vertical displacement of the boat, are shown with respect to fixed inertial axes rather than with respect to the water surface. If the boat was undisturbed by the waves, therefore, the values of α_0 and z_0 would be zero. If the center of gravity of the boat moved exactly with the surface of the water, the value of α_0 would be 0.01 and z_0 would equal z_w. Because of the sign conventions assumed (vertical motion of the boat positive downward, and vertical velocity

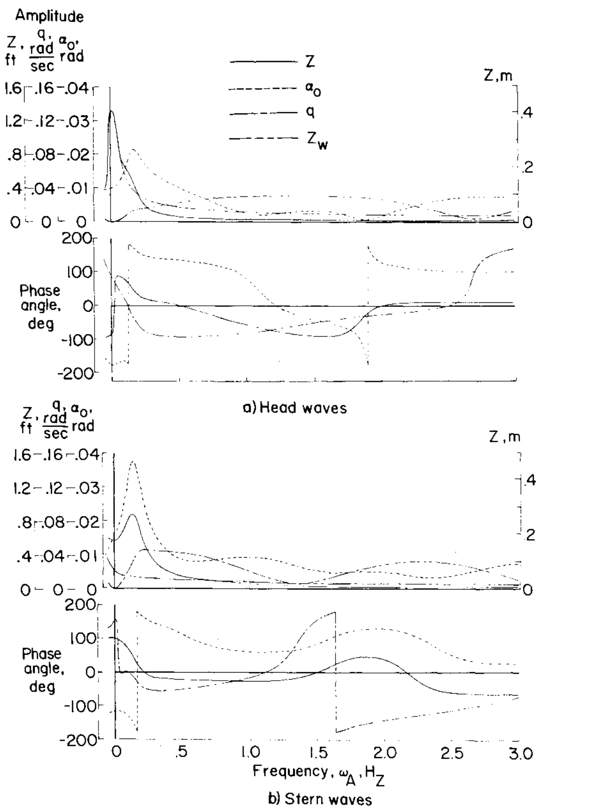

Fig. 8 **Frequency response to sinusoidal waves, $\alpha_{\omega_0} = 0.01$ rad, standard case, $H_z = 47.5$ N-m/m (10.7 ft-lb/ft).**

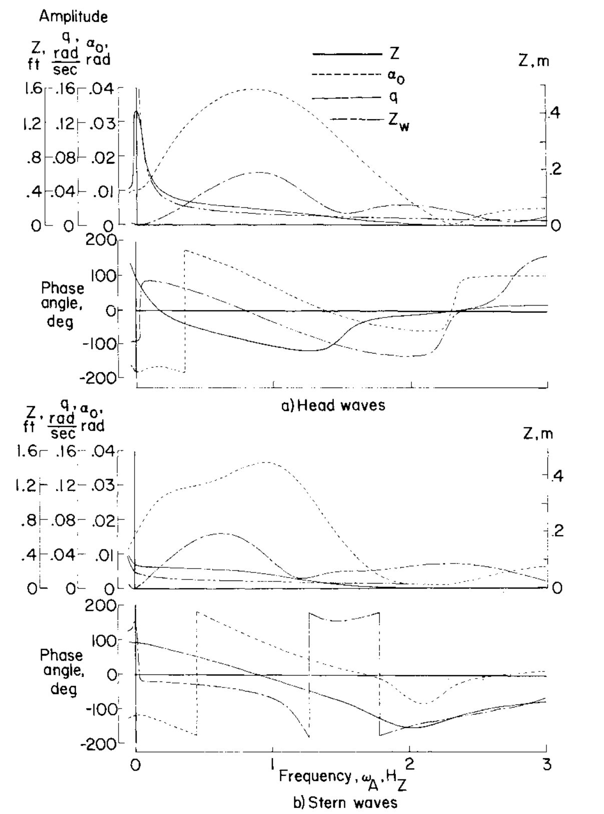

Fig. 9 **Frequency response to sinusoidal waves, $H_z = 222$ N-m/m (50 ft-lb/ft), other variables standard.**

of the water positive upward) the phase angle of α_0 would be $\pm 180°$ and the phase angle of z_0 would be $+90°$.

A peculiarity in the frequency response curves occurs at low frequencies or long wavelengths because of the assumption of deep water. For head waves, the frequency approaches zero as the wavelength approaches infinity because the wave speed varies as the square root of the wavelength and,therefore, influences the frequency to a decreasing extent as the wavelength approaches infinity. For stern waves, the wave speed equals the boat speed at some finite wavelength, causing the frequency to go to zero. At longer wavelengths, the waves overtake the boat and the frequency increases again to some value, but subsequently decreases to zero as the wavelength goes to infinity. The response of the boat to stern waves is, therefore, a triple-valued function of frequency in the low-frequency range. This behavior is illustrated in Fig. 10, which shows the actual frequency, ω_A, as a function of ω_B, the frequency encountered by the boat in moving over a fixed, wavy surface with the same wavelength as the waves on the water. The corresponding values of wavelength are also shown. Though the frequency in the region where the waves are overtaking the boat is a real physical quantity, it is convenient to plot this frequency as a negative value to avoid confusion on the plots. The plot for head waves, therefore, includes in the negative frequency range the region for stern waves running from $\omega_A = 0$ to the first maximum of ω_A, and the plot for stern waves includes in the negative frequency range the region for stern waves running from the first maximum of ω_A to the point where ω_A returns to zero. In practice, these regions may be of little significance because the actual wave spectrum may not contain such long-wavelength components.

An important factor in evaluating the riding qualities of a boat is the normal acceleration resulting from riding over waves. The normal acceleration at the center of gravity as a function of frequency for head waves and stern waves and for values of H_z of 47.5 and 222 N-m/m is shown in Fig. 11. These values are obtained as the second derivative of the ver-

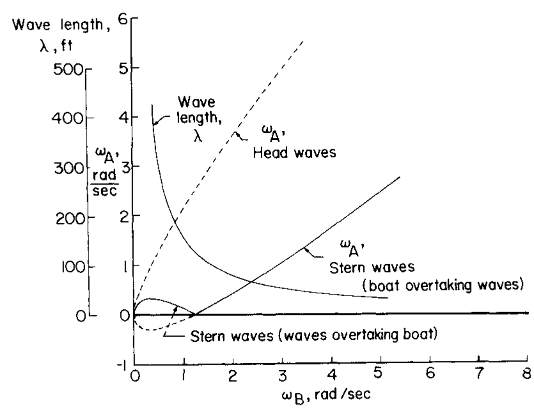

Fig. 10 **Variation of actual frequency ω_A with ω_B for head waves and stern waves in deep water. Corresponding values of wavelength λ are shown. $\omega_B = 2\pi V/\lambda$, $V = 7.62$ m/sec (25 fps).**

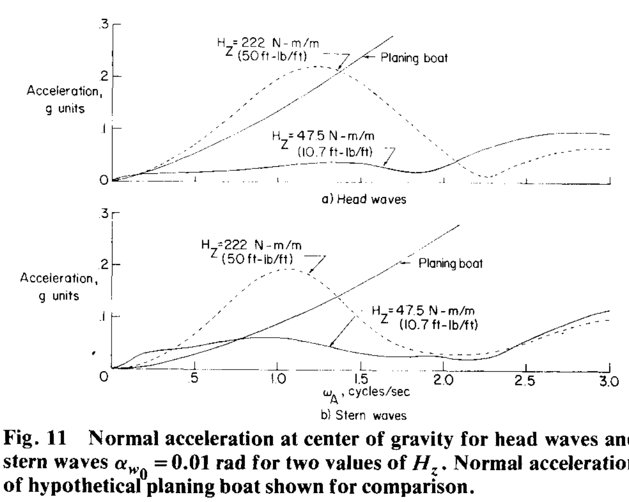

Fig. 11 **Normal acceleration at center of gravity for head waves and stern waves $\alpha_{w_0} = 0.01$ rad for two values of H_z. Normal acceleration of hypothetical planing boat shown for comparison.**

tical displacement. The normal acceleration which would be experienced by a hypothetical planing boat exactly following the surface of the water is shown for comparison.

Discussion

The root locus studies show that adequate stability of all modes of motion may be obtained with a system of the type analyzed. The predominant low-frequency mode of the boat, however, appears to have a frequency too low to interact with the bobweight system on the flap. The damping of this rigid-body mode is derived primarily from the lift and pitching moment derivatives of the boat. The bobweight contributes to the damping of the higher frequency modes involving primarily flap motion. The bobweight, however, contributes most of the inertia of the flap system. If the bobweight with its high inertia were removed, the flap modes would probably have an even higher frequency and adequate damping. The combination of low flap inertia and no bobweight was not studied in this investigation. The original premise that the bobweight would contribute to the damping of the rigid-body motion of the boat, however, was found to be incorrect.

A large difference was found in the response to head waves and stern waves. This difference had been observed previously in experimental studies of hydrofoil boats (Ref. 5, for example). The system studied with a relatively low value of the variation of restoring force with vertical displacement ($H_z = 47.5$ N-m/m) provides excellent attenuation of the vertical motions of the boat due to head waves throughout a large range of frequencies, whereas in stern waves the motion is attenuated to a value less than the wave amplitude at frequencies above 0.8 Hz but amplified at frequencies below this value. The system with a larger value of restoring force ($H_z = 222$ N-m/m) has larger response to both head waves and stern waves except in stern waves at very low frequencies.

The larger disturbing effect of stern waves is accounted for by the pitching moment applied to the boat due to the difference in vertical velocity of the water at the front and rear foils. As shown in Fig. 9, a much shorter wavelength is required to produce a given response frequency ω_A for stern waves than for head waves. For a given wave height, this shorter wavelength, in general, produces a larger difference in angle of attack between the front and rear foils because the boat length extends over a greater fraction of the wavelength.

In more sophisticated control systems for hydrofoil boats, such as that of Ref. 6, a pitch attitude gyro is used in conjunction with other sensors to provide an inertial reference for stabilizing the pitching motion of the boat. The control system studied herein, which uses height and normal acceleration sensors and which has no inherent stiffness to resist pitch disturbances, would not be expected to be very effective in offsetting the relatively large pitch disturbances caused by low-frequency stern waves. Possibly, in restricted bodies of water, however, the amplitude of waves of the larger wave lengths would be quite small. Studies of the spectra of waves likely to be encountered by a small boat in restricted bodies of water would be desirable to allow calculation of actual response amplitude as a function of frequency.

Conclusions

The longitudinal control system studied is capable of providing adequate stability and smooth return to an equilibrium condition following disturbances. The acceleration sensor or bobweight does not contribute appreciably to the stability of the rigid-body motion of the boat. The system with a relatively low restoring force as a function of vertical height gives the smaller response to waves over most of the frequency range. The response to head waves appears to be desirably small, but the response to stern waves at low frequencies is larger than desired.

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